Long-Range Monitoring of Fuel Consumption of Car Based on Generalized Spectral-Analytical Method

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***Abstract* — Proposed is the approach to the analysis of fuel consumption over a long lifetime of a vehicle according to the facts of refueling and car mileage. This case study may serve as an example of long-range analysis of cumulative indicators of the car and the driver.**

Key Words — cumulative sum, long-range difference, orthogonal bases

1. Introduction Variables

Consider the problem of determining the average fuel consumption over a period of operation of the vehicle. Such an analysis allows us to consider seasonal variations in fuel consumption, the effect of driving style on fuel consumption, the change in fuel consumption over the life of the vehicle, in particular, to identify periods of running or aging car.

Assume that s - car mileage, g(s) - instant fuel consumption, G(s) - total fuel consumption, and holds:

$$G(s) = G(s_0) + \int_{s_0}^{s} g(s)ds \qquad (1)$$

The challenge is that for certain *G(s)* to find *g(s)*. Real input data is a table of values of the samples t_i, s_i, G_i. Time *t* can also serve as independent variable in the equation (1) in the case of determining fuel consumption per time.

Table 1 shows such a table - log of refills - for a continuous period of vehicle operation.

2. Solution of the problem

The general solution of the problem lays in the differentiation of the cumulative sum of all refills *G(s)*:

$$g(s) = \frac{dG(s)}{ds} \qquad (2)$$

The simplest method of differentiation is to divide the finite difference

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$$g(s) = \frac{G_{i+1} - G_i}{s_{i+1} - s_i} \qquad (3)$$

However, a more stable solution for differentiation of a cumulative function is given by

$$g(s) = \frac{G_{i+n} - G_i}{s_{i+n} - s_i} \qquad (4)$$

Equation (3) is a special case of (4) for *n=1*. The formula (4) is a method of obtaining flow function which is more resistant to fluctuations. Equation (4) utilizes long-range differences.

Tab 1. Simple registration of refueling and mileage of vehicle on petrol station.

Num	Day	Month	Year	Mileage (km)	Fuel Filling (l)
1	24	11	2007	20	40
2	1	12	2007	300	40
3	20	12	2007	950	40
…	…	…	…	…	…
86	5	12	2010	45800	41
87	22	12	2010	46200	45
88	6	1	2011	46800	40

A further complication of the method is based on the expansion of the cumulative function in a series of orthogonal polynomials, and applying analytical differentiation [1,2,3]:

$$G(s) = \sum_{i=0}^{N} c_i T_i(s) \qquad (5)$$

The main requirement to an analytical data approximation is an imperative fulfillment of the condition $N = N_{min}$, which means that the number of terms in the truncated orthogonal series should be minimum among all orthogonal bases in providing a given uniform inaccuracy of data description. This condition is fulfilled if we introduce adaptive procedures into computations of expansion coefficients, namely, the choice of basis and its parameters. It should be noted that introduction of adaptation procedures into the process of the expansion coefficients calculation not only provides fulfillment of the condition $N = N_{min}$, but also assists regularization of the conditions when solving inverse ill-posed problems such as differentiation.

In the course of an analytical data description we calculate the expansion coefficients in terms of a chosen basis that is we calculate a spectrum of orthogonal functions of a specific orthogonal basis. An analytical data description in the form of truncated orthogonal series is used in analytical transformations and derivations in order to obtain necessary estimates and characteristics in different application problems [3,4,5,6,7].

3. Results

Figure 1 shows the version of the code that performs all necessary calculations by formula (4) with $n=6$, and Figure 2 the corresponding chart of the desired function. Figure 2 also shows the results of processing the data using the analytical description (5) and analytical derivation using the Chebyshev polynomials of the first kind for $N=30$. Spectral algorithms of Chebyshev approximation and derivation of Chebyshev approximated function are described in [2].

```
data=Import["data11184.txt","Table"];
km=data[[All,4]];
lt=data[[All,5]];
lt=Accumulate[lt];
g[0]:={km/1000,lt 100/km}//Transpose//N;
g[n_Integer]:={Drop[Drop[km,-n/2],n/2]/1000,(Drop[lt,n]-
Drop[lt,-n])/(Drop[km,n]-Drop[km,-n])
100}//Transpose//N;
ListPlot[gr[6],Joined->True]
```

Fig. 1: Code Listing for Wolfram Mathematica. The corresponding output plot is on the figure 2.

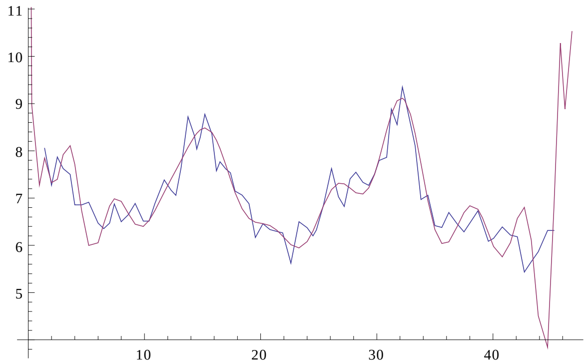

Fig.2 Fuel consumption (litres per 100 km) versus car mileage (thousands of kilometers) for LADA KALINA 1,4 16 valve (VAZ11184) during three-year observation according to the long-term difference (blue curve) and spectral (brown curve) averaging procedures.

Both curves on figure 2 agree well and show a seasonal variation in fuel consumption. Note that the analytic curve shows a loss of accuracy at the edges.

4. Conclusion

The study showed that a simple solution using the ratio of the long-range differences gives a good approximation to the solution of the problem, as well as solution based on the analytical approximation.

Note that the approaches in this paper are based on a simple registration of counter readings at times of filling the car. This procedure was carried out manually, but it would be convenient to equip on-board car computer with such a similar function.

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