Recognition of Lines Detected in the Image Plane on the Basis of the Generalized Spectral-Analytical Method

R. K. Tetuev

Institute of Mathematical Problems in Biology, Russian Academy of Sciences, ul. Institutskaya 4, Pushchino, Moscow oblast, 142290 Russia

e-mail: radja@imnb.ru

Abstract—In recognizing visual images, it is important to have a quantitative characteristic of similarity between lines detected in the image plane (such as closed contours, gradient discontinuities, curves, etc.). To obtain such a characteristic, we vectorize each line and find the dependence between the direction angle of its traversal vector and the traversed path length (the course function). The generalized spectral-analytical method (which consists in completely processing data in the space of the Fourier coefficients obtained by expanding the course functions in orthogonal series) provides means for fast estimation of the similarity of lines with various locations, sizes, orientations, symmetries, etc. The effectiveness of this approach is tested in the example of Chebyshev polynomials (of a discrete argument).

INTRODUCTION

In solving problems related to recognition of visual images, it is often necessary to compare the shapes of various lines detected in the plane of some image (such as closed contours, gradient discontinuities, curves, etc.) and quantitatively estimate the similarity between these lines.

When the amount of the processed data is very large, it is especially important to develop a method that requires modest computer resources (both time and memory).

In this connection, we suggest to solve this application problem by the generalized spectral-analytical method with the use of orthogonal bases of polynomials in a discrete argument. We choose this method because the algorithms based on it have a high speed of data processing and a small amount of memory needed for storing the main expansion coefficients.

In this paper, we use the theoretical approach described above for solving a special case of the problem of quantitatively estimation of the similarity between nonclosed curves formed by points in given images.

First, we vectorize the given groups of points, i.e., take a sample $\{(X_i, Y_i)\}$, where $0 \le i \le L$, of an arbitrary size L from the group of points belonging to the given line; the sample should describe the behavior of the given line with sufficient precision (see Fig. 1).

Next, we construct the array of transition vectors $\bar{V}_i$ from the ith point in the sample to the $(i+1)$th point for each $\{\bar{V}_i\}$: $\bar{V}_i = \overline{(X_{i+1} - X_i, Y_{i+1} - Y_i)}$.

Clearly, the quantity

$$S = \sum_{i=1}^{L} \sqrt{(X_{i+1} - X_i)^2 + (Y_{n+1} - Y_n)^2}$$

is approximately equal to the length of the line, i.e., the traversed path.

We introduce the variable

$$s_i = \sum_{n=1}^{i} \sqrt{(X_{n+1} - X_n)^2 + (Y_{n+1} - Y_n)^2},$$

which equals the traversed path.

Now, let us determine the dependence of the direction angle of the traversal vector $\bar{V}_i$ in the form

$$W(s_i) = \arctan\left(\frac{Y_{i+1} - Y_i}{X_{i+1} - X_i}\right).$$

This function $W(s_i)$ (the course function) is defined on the generally nonuniform discrete grid $0 \le s_i \le S$.

Let us transfer the function $W(s_i)$ to a uniform grid of fixed size T, which is the same for all lines under con-

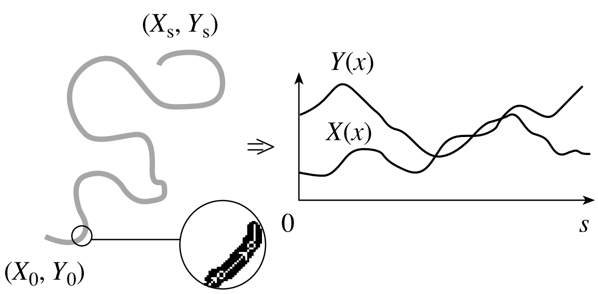

Fig. 1.

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sideration, by applying the transformation $0 \le t = s_i T/S \le T$ for $0 \le i \le L$ and denote the obtained function by $\overline{W}(t)$ (see Fig. 2).

In what follows, we use the generalized spectral-analytical method, i.e., assume that the data are completely processed in the space of Fourier coefficients $\{C_k\}$, $0 \le k \le N \le T$, which are obtained by expanding the course functions $\overline{W}(t)$ in modified classical orthogonal polynomials or in functions of continuous and discrete arguments. This makes it possible to construct criteria for a fast and effective quantitative estimation of the similarity and other relations between the lines under consideration, which is performed entirely in the space of coefficients. In applied problems of this kind, similar images are differently arranged and have different sizes, orientations, symmetries, and other characteristics.

GEOMETRIC RELATIONS BETWEEN AND OPERATIONS ON LINES IN THE SPACE OF FOURIER COEFFICIENTS

Note that, to obtain a sufficiently accurate vector representation of lines, we need only the first several Fourier coefficients $\{C_k\}$ of the function $\overline{W}(t)$, S (the length of the traversed path of the line, which depends on the line size), and the coordinates of, say, the first point (X_0, Y_0) (which depends on the location of the line and the traversal direction):

$$\tilde{X}_j = X_0 + S/T\sum_{t=0}^{j}\cos\overline{W}(t) \approx X_0 + S/T\sum_{t=0}^{j}\cos\left(\sum_{n=0}^{N}C_n f_n(t)\right),$$

$$\tilde{Y}_j = X_0 + S/T\sum_{t=0}^{j}\sin\overline{W}(t) \approx Y_0 + S/T\sum_{t=0}^{j}\sin\left(\sum_{n=0}^{N}C_n f_n(t)\right),$$

where the $f_n(t)$ are the basis functions.

Thus, we obtain an alternative vector representation $\{(\tilde{X}_j, \tilde{Y}_j)\}$ $(0 \le j \le T)$ of the line, which is geometrically close to the initial representation $\{(X_j, Y_j)\}$.

The above considerations suggest a number of important properties common to all orthogonal systems used.

Suppose that two lines have path lengths S and $\tilde{S}$ and Fourier coefficients $\{C_k\}$ and $\{\tilde{C}_k\}$. Then the following properties hold.

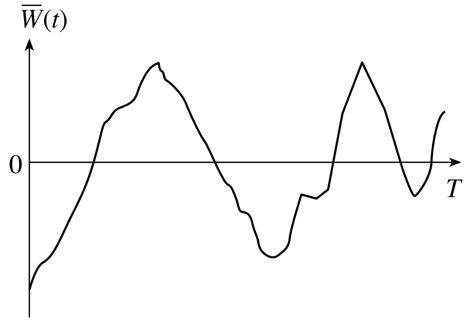

Fig. 2.

Property 1. The ratio of the sizes of the lines is approximately equal to $S/\tilde{S}$.

Property 2. For lines of close shapes but different in location and size,

$$C_k \approx \tilde{C}_k.$$

Property 3. For mirror-symmetric lines of close shapes,

$$C_k \approx -\tilde{C}_k.$$

Property 4. For lines of close shapes with different spatial orientations,

$$C_k \approx \tilde{C}_k \quad \text{if} \quad k \neq 0$$

and

$$\omega \approx (\tilde{C}_0 - C_0)/T,$$

where ω is the angle between the lines.

Note that the Fourier coefficients $\{C_k\}$ and $\{\tilde{C}_k\}$ found for the same line traversed in opposite directions, i.e., for $\tilde{W}(s_i) = W(S - s_i)$, are generally different. But if symmetric orthogonal systems are used (i.e., each basis function is either even or odd with respect to the middle point of the interval of definition), then they have the following additional properties.

Property 5. If a symmetric basis is used, then the coefficients $\{C_k\}$ and $\{\tilde{C}_k\}$ of the course functions $\overline{W}(t)$ and $\tilde{W}(t)$ of the same line traversed in opposite directions (i.e., such that $\overline{\tilde{W}}(t) = \overline{W}(T - t)$) satisfy the relation

$$C_i \approx \tilde{C}_i,$$

where i is the number of an even basis function, or

$$C_j \approx -\tilde{C}_j,$$

where j is the number of an odd basis function.

Property 6. Under the conditions of Property 5, according to Properties 3 and 6, any Fourier coeffi-

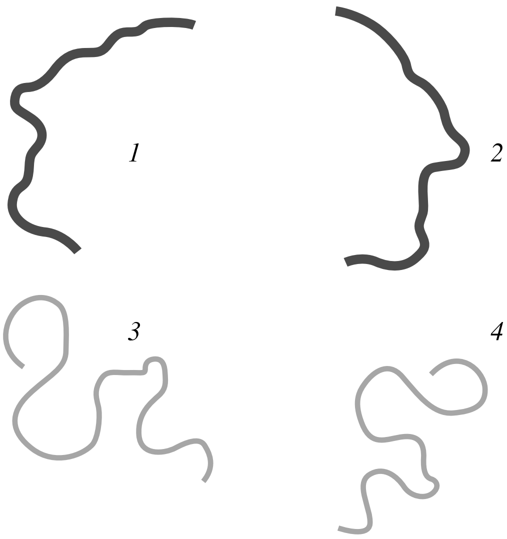

Fig. 3. The lines of the curves under consideration.

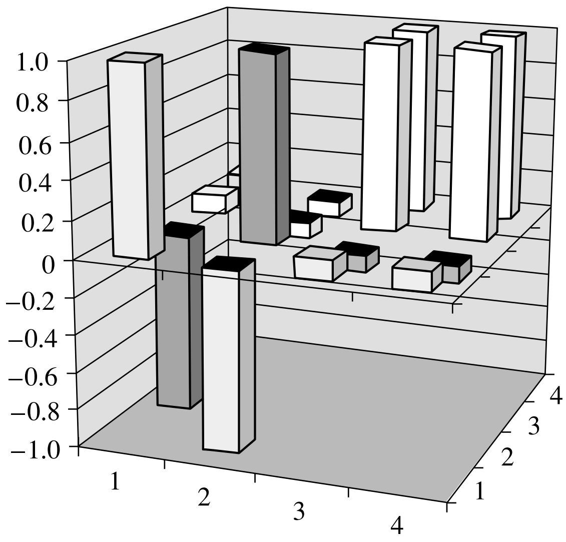

Fig. 4. Correlation coefficients.

cients $\{C_k\}$ of mirror-symmetric lines satisfy the condition

$$C_j \approx 0,$$

where j is the number of an odd basis function.

Property 7. Under the conditions of Property 5, for centrally symmetric lines,

$$C_i \approx 0,$$

where i is the number of an even basis function.

The basis functions are even (odd) for the systems of trigonometric functions, the Legendre and Chebyshev polynomials, and many other orthogonal bases.

According to the revealed properties, we can perform the basic geometric operations on lines in the space of coefficients by using elementary mathematical transformations of their coefficients, which improves the performance of the computer implementations of the algorithms based on the proposed approach.

Table

	Line 1	Line 2	Line 3	Line 4
C0	–926	–674	2010	–2432
C1	–485	428	134	134
C2	–80	61	–127	–127
C3	–65	49	415	414
C4	–56	46	–3666	–367
C5	–45	5	–246	–247
C6	121	–149	306	307
C7	125	–74	246	248
C8	7	81	51	50
C9	–42	50	–27	–28

The properties listed above are used to construct criteria for quantitatively estimating the basic geometric properties of and relations between the lines under consideration, such as similarity, mirror symmetry, central symmetry of a line, mutual arrangement, etc. For example, a practically convenient criterion for estimating the closedness of the shapes of two different lines is the 3
correlation between the corresponding series of coefficients $\{C_k\}$ and $\{\tilde{C}_k\}$. In this case,

(i) a correlation coefficient close to 1 is interpreted as the high similarity of the lines;

(ii) a correlation coefficient close to 0 is interpreted as an insignificant similarity between the lines;

(iii) a correlation coefficient close to –1 is interpreted as the mirror symmetry of two similar lines.

RECOGNITION OF CURVES

The theoretical approach described above was applied to solve the special case of the applied problem under consideration in which the initial images contain groups of points that form nonclosed curves. Below, we present the results yielded by an algorithm implementing this approach. The program was fed by the images of curves shown in Fig. 3. As the orthogonal system, the Chebyshev polynomials in a discrete argument defined on a grid of size $T = 100$ were used. The distances were measured in pixels, and the angles were measured in degrees. The criterion for the closedness of the expansion coefficients was the correlation coefficient of the first $N = 10$ terms except C_0 (by virtue of Property 4).

The expansion coefficients for the given curves are presented in the table.

It is seen that the correlation coefficients are equal to 1 on the main diagonal of the matrix, and the elements symmetric about the diagonal are equal (Fig. 4). It is interesting to note that the element 2×3 is close to 1, the element 1×2 is close to –1, and the elements 1×3, 1×4, 2×3, and 2×4 are close to 0; according to the theoretical data, this means the similarity, similarity with symmetry, and absence of any significant similarity between the corresponding lines.

CONCLUSIONS

The theoretical approach suggested above is very promising and convenient, because it naturally makes it possible to solve complicated multidimensional recognition problems by classical methods of orthogonal expansions, which have extensive theoretical base and research material.

REFERENCES

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