Contour Recognition Based on Spectral Methods. Solution of the Problem of Choice of the Start-Point

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Abstract—The paper is devoted to the use of spectral methods in problems of visual pattern recognition. The main idea is to associate a two-dimensional closed line, treated as a univariate function, with the contour of each object. On the basis of analysis of expansion coefficients of these functions, we propose adequate quantitative estimates for similarity of objects, which are invariant under affine transformations of the plane. A particular result is the invariance of the spectral representation with respect to the choice of the start-point. This invariance is obtained on the basis of the sine–cosine decomposition of arbitrary periodic functions.

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INTRODUCTION

Many researchers agree in opinion that the silhouette of an object is a more informative part of the pattern[1] than its color. By virtue of conditional character of this proposition, mathematicians solving the problem of pattern recognition unintentionally participate in one of the crucial disputes of fine arts, namely, the dispute about "color and shape." However, in contrast to artists, mathematicians are guided by trusted concepts of effectiveness and stability of quantitative estimates for similarity of images[2] rather than by aesthetic preferences and adequacy of the attitude.

For instance, it is clear that the color of a tomato is much more informative for determining the ripeness of the object (tomato) than its shape. However, in sorting, for instance, eggplants intermixed with squashes, the difference in shapes can be quite useful.

We emphasize that there are problems dealing initially only with the concept of a shape, where no filters[3] for images are needed. Problems such as recognition of an object knowing only its shadow operate only with a mask[4] of an object without any other information. However, the recognition of "colored" visual objects if also often naturally reducible to the problem of distinction of figures[5], typical marks, contrast pictures, etc. In this case, the differences in colors may play the role of auxiliary information.

In this paper, we discuss pattern recognition which is performed exclusively on the basis of distinction of silhouettes, contours[6] determining the pattern shape, but not the internal points[7] of the pattern.

We clarify that the filters should be chosen in such a way that objects of the nature interesting for us in the plane are well distinguishable in the plane of the image mask. Depending on the application area, quite complex filters are sometimes needed which help to analyze the loci of discontinuities of color gradients of the image, calculate fractal dimensionalities, etc.

Generally speaking, the discussion about the design of such filters is beyond of the scope of this paper. However, many recognition problems, to which researchers have at last drawn their attention, can be surprisingly easily filtered with simple filters and decomposed into a variety of typical figures.

In the next section, we develop a method, using which one can easily and quickly describe these figures and compare them with one another.

GENERAL RECOGNITION SCHEME. METHOD OF RADIUS FUNCTION

Recognition of visual patterns by a recognition system must be fulfilled in the automatic mode in accordance with a unique scheme (common for all objects) designed by a certain recognition method. In this section, we describe a scheme based on distinction of the

[1] **A pattern** is a group of points of an image determined by one object.

[2] **An image** is a two-dimensional array of multicolored points (pixels).

[3] **A filter** (here) is a Boolean function of the image point (object/nonobject).

[4] **A mask** (here) is the result of the action of a filter on all points of an image.

[5] **A figure** (here) is a simply connected closed domain of the mask.

[6] **A contour** (here) is a group of boundary (not internal) points of the figure.

[7] **An internal point** (here) is a point such that all its neighboring points have the same color.

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spectral representation of one of the most important components of visual patterns.

In the general case of a color image, we use a certain filter in order to obtain a mask (a monochromatic black and white picture). Individual groups of black points are associated with the pattern of an object of nature interesting for us. It is assumed that similar objects are characterized by the similarity of their shapes (figures) in the geometrical sense. Of course, we cannot expect that the silhouettes coincide exactly, pixelwise because of natural distortions of color rendition, different scaling, translations, and rotation angles of patterns in the plane. These are the reasons why we must apply methods to a great degree approximate, which are stable under noise actions.

Thus, in the simplest case, we have two black "spots" on the white background, and we have to estimate quantitatively the similarity of their shapes. It is natural to propose (for saving in computer time and memory) to compare the relative positions of boundary points rather than all points. The fact that, among the nearest neighbors of any boundary point of the figure, there are just two points which are also boundary points allows us to design a process for figure tracing such that all boundary points become ordered after this process. During the tracing process, we should proceed in accordance with a certain agreement about the direction of the motion: clockwise or counterclockwise.

This process (referred to as vectorization, see [2]) being completed, we obtain an ordered set of coordinates of the boundary points, which may be represented in the form of two arrays $\{X_i\}$ and $\{Y_i\}$, where $1 \le i \le N$ and N is the number of all boundary points of the given figure. It is clear that the arrays obtained may be represented as discrete values of some periodic functions. However, it is obvious that, taking different boundary points of the same figure as the start-points of vectorization, we obtain, in the general case, different coordinate arrays shifted with respect to each other. One often tries to resolve this indeterminacy (the so-called problem of the start-point choice) by taking the most distant point from the mass center[8] of the contour or even figure as the start-point. Below, we will also use this approach except for one explicitly specified case (see *Spectral Invariants of the Choice of the Start-Point. The Clock-Diagram Method*).

Since the distance between a given point of the contour and its mass center is independent of transition and rotations of the figure in the plane, we propose to pass from Euclidean coordinates (X_i, Y_i) to polar coordinates (R_i, ω_i) with the origin

$$(X_0, Y_0) = \left(\frac{\sum_{i=1}^{N} X_i}{N}, \frac{\sum_{i=1}^{N} Y_i}{N}\right).$$

Note that the form of the function of radius-component $\{R_i\}$ of tracing vector (R, ω) is expressible and easily distinguishable for most figures. However, this does not hold for the azimuth-component $\{\omega_i\}$; therefore, in the recognition scheme below, the latter is not involved. This significantly simplifies and accelerates the process in practice.

At this stage, for each contour we obtain an array $\{R_i\}$. The size of this array is quite large and different for different figures. Therefore, it is difficult to estimate the similarity directly. It is much better to represent $\{R_i\}$ in the form of orthogonal expansiuon[9] in some predefined functions $\{\varphi_i(t_i)\}$

$$R_i = \sum_{n=0}^{N-1} C_n \varphi_n(t_i).$$

Actually, the orthogonal expansion transforms each array $\{R_i\}$ into an array of expansion coefficients $\{C_i\}$ of the same size N. However, to compare figures approximately, it is sufficient to use only a few first $k < N$ coefficients that provide a satisfactory approximation[10] of the array $\{R_i\}$:

$$R_i \approx \sum_{n=0}^{k-1} C_n \varphi_n(t_i).$$

As the system of functions $\{\phi_n(t_i)\}$ for such a representation, it is convenient to specify the set of discrete Tchebychev polynomials $\{\theta_n(t_i)\}$, which are orthogonal with the unit weight on the uniform grid $t_i = i$ of arbitrary size $0 < i < N - 1$ (see [1]). This is the orthogonal basis that is used in most examples in the paper.

To realize the automatic estimation of the figure similarity, it remains only to propose a quantitative estimate for similarity of coefficients $\{C_n\}$ determined by each contour. As has been mentioned above, we should apply methods, which are sufficiently approximating and stable under the noise action. These requirements are totally met by the estimation of the correlation of the corresponding expansion coefficients of different contours. Then, numerical results for contours with

[8] **The mass center** is the point whose coordinates are the arithmetic means of the corresponding coordinates of the group of points.

[9] **Orthogonal expansion** is a representation of the function in the form of a linear form of a series of functions, which compose a system (spectrum).

[10] **An approximation** (here) is an approximate representation of the function in the form of an incomplete orthogonal expansion.

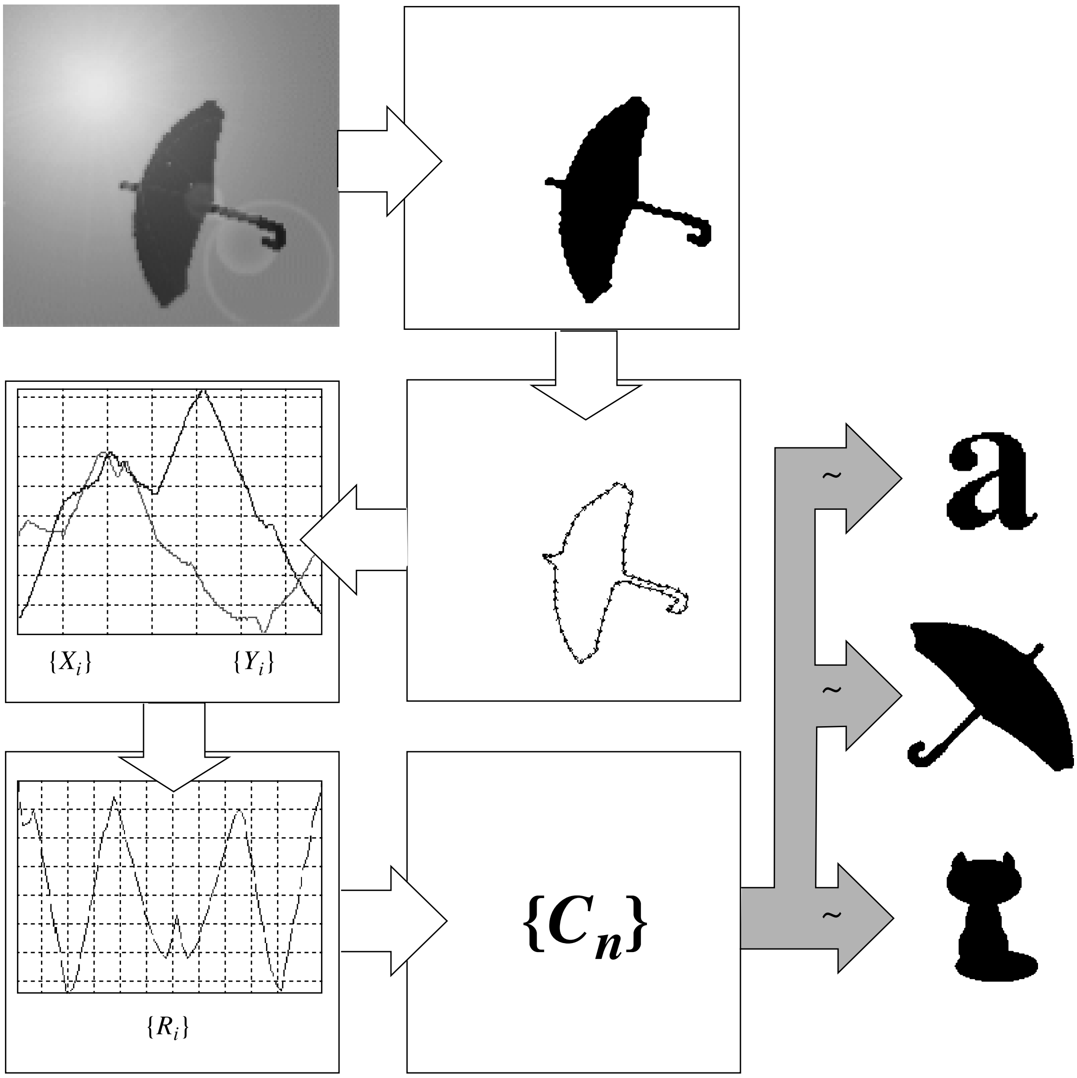

Fig. 1. Stages of pattern recognition based on distinction of contours.

coefficient series $\{C_n\}$ and $\{C_n'\}$ may be easily interpreted as follows:

In the case of almost zero correlation

$$\rho(\{C_n\}, \{C_n'\}) \approx 0,$$

we obtain the absence of the correlation between the corresponding objects. However, we have a high probability of similarity in the case

$$\rho(\{C_n\}, \{C_n'\}) \approx 1.$$

As a result, we have obtained a fast and efficient method for pattern recognition based on analysis of object contours. We will refer to it as the method of radius function. We write the main stages of this method in the following form:

• Conversion of the visual pattern into the space of coefficients:

– construction of the mask of the object;

– extraction of the figure and vectorization of the contour;

– passage to the polar coordinate system;

– calculation of the expansion coefficients of the radius function.

• Quantitative evaluation of similarity of objects based on analysis of the correlation of the corresponding expansion coefficients.

The main stages of the recognition scheme are illustrated in Fig. 1.

APPLICATION OF THE METHOD OF RADIUS FUNCTION TO CHARACTER RECOGNITION

To demonstrate the efficiency of the method discussed, we perform a numerical experiment by testing this method on an example of recognition of arbitrary symbols. Note that the recognition, as an intelligent

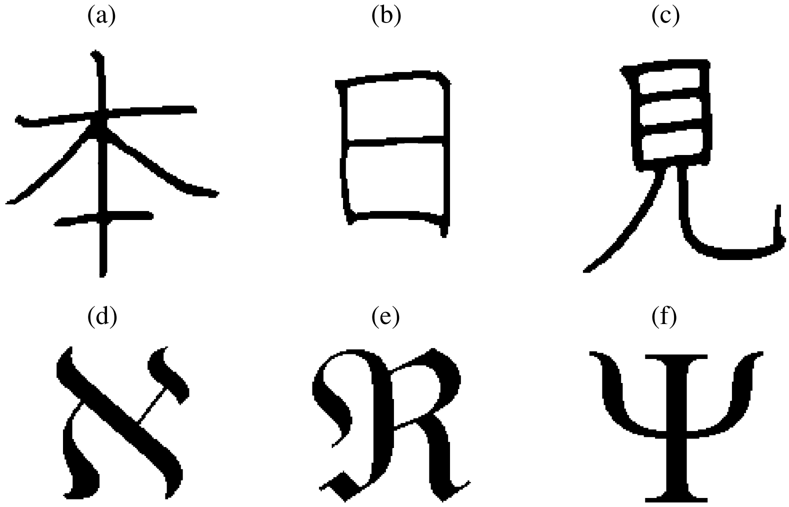

Fig. 2. A collection of writing symbols.

process, may be represented in the most general sense by two individual phases.

The first phase consists in active accumulation of information. During this phase, the system of pattern recognition, in accordance with a predefined scheme, processes and analyzes the information about all objects presented to it. On the basis of this information, it generates its own representation for each presented object. These representations are stored in a repository, a data bank. Each record in this bank is also associated with our understanding of the object. The first phase is often referred to as the learning phase of the recognition system.

The second phase begins with the presentation of a new unknown object to the system. First of all, the object is processed by the same scheme that has been used by the system at the learning stage for processing the known objects. Then, the obtained representation of the new object is used for retrieving, in the data bank, a record about an object whose representation, in a sense, is closest to this object. As a result, if the system acknowledges an object of this type, then we obtain the matching between the pattern of the new object and one of the objects used by the system at the learning stage.

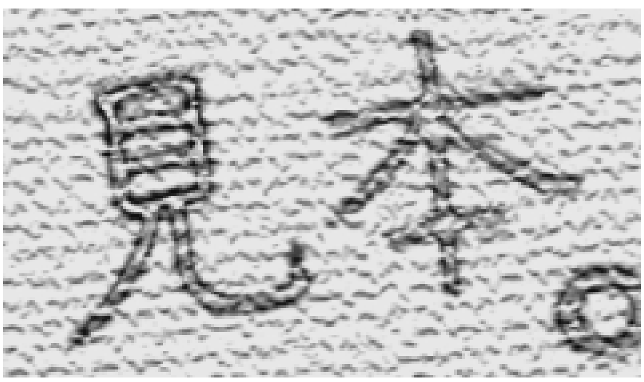

Fig. 3. A sample of Kanji hieroglyphic writing.

The second phase is often referred to as identification of the object.

Thus, let us take a system of pattern recognition, which, for the sake of simplicity, have representations of only three Kanji hieroglyphs and three letters from different alphabets (see Fig. 2). The representations are generated by the system in accordance with the scheme discussed in the previous section; i.e., each pattern is associated with a tuple of, for instance, twenty numbers, which are the first coefficients of the expansion of the radius function of the contour.

Next, we provide the system with three objects for identification, which are extracted from the images in Figs. 3 (symbols 1 and 2) and 4 (symbol 3).

As a result of processing the symbol contours, both used for learning and intended for identification, we obtain the spectral representation of their radius functions in the form of series of the expansion coefficients (Table 1).

As has been said above, the system estimates the similarity of the objects on the basis of the correlation between the series of coefficients. By matching the representation of the new objects with those known by system since the learning stage, we obtain quantitative estimates for the similarity. These estimates are presented in Table 2.

Analyzing Table 2, we can easily conclude that symbols 1 and 2 are, with a high probability, the Kanji hieroglyphs that denote the words "look" and "book," respectively, while the tribal mark (symbol 3) originates most probably from the Greek capital letter "psi." We see that the computer results obtained in this numerical experiment are to a high degree adequate to decisions made by most people in this situation. This confirms the applicability of this approach to problems of pattern recognition.

SPECTRAL INVARIANTS OF THE CHOICE OF THE START-POINT. THE CLOCK-DIAGRAM METHOD

Let us go back to discussion of the uncertainty of the choice of the start-point that arises at the stage of vectorization mentioned above in the section "General Recognition Scheme. Method of Radius Function."

First, for convenience, we introduce the notation

$$\{t+\theta\}_T = T\{(t+\theta)/T\},$$

where $\{\tau\}$ is the fractional part of number τ.

Let $\phi(t)$ and $\psi(t)$ be some functions defined on the interval $(0, T)$. If there exists a unique $0 < \theta < T$ such that

$$\phi(t) = \psi(\{t+\theta\}_T),$$

then we say that functions $\phi(t)$ and $\psi(t)$ are equal in period T (or simply "are equal in period") and the quantity θ will be referred to as the phase shift. If there exist N values $\theta_1, \theta_2, \ldots, \theta_N$ for which this equality holds, then functions $\phi(t)$ and $\psi(t)$ are equal in period T/N (or multiply equal in period).

Now, we formulate the problem of the start-point choice in the general form as follows;

it is required to find the phase shift for two arbitrary functions if these functions are equal in period or prove that this proposition does not hold.

Of course, in practice we should replace the exact coincidence by an approximate one

$$\phi(t) \approx \psi(\{t+\theta\}_T)$$

and the uniqueness by an extremum. In this case, we will speak about functional closeness in period.

All attempts to solve this problem may be divided into two types. Those of the first type are aimed at the design of a fast and efficient tool for numerical calculation of the unique (stable) value of the phase shift, while the attempts of the second type are aimed at the search for invariant quantities, which are independent of the phase shift and, at the same time, may be used for recognition of periodic functions.

In practice, the first approach often results in unreasonably high expenses of resources, while the second approach is more elegant but has obvious drawbacks. The reason is that the group of invariants is not a complete system of features; i.e., knowing all such features, we cannot uniquely reconstruct the function. This means that, even in the case of full coincidence of all invariants of the functions, the system may assert only that it is "very probable" that the functions are "very similar."

Therefore, it seems very attractive to derive a rigorous stable representation of periodic functions, which is absolutely independent and invariant with respect to the phase shift, together with the phase shift θ_0 relative to a certain reference position. In this case, knowing the representation and quantity θ_0, we can always reconstruct the function. This would allow us to assert that the periodic functions are close in shape to the same degree as their representations are close and that the difference of the corresponding values of rotations with respect to a reference position (the difference of values of θ_0) is equal to the phase shift of these functions with respect to each other.

Table 1. Representation of the radius function by the coefficient series

	C_0	C_1	C_2	C_3	…
A	–0.09	0.09	0.02	–0.13	…
B	–0.13	0.27	–0.04	–0.20	…
C	0.16	0.25	–0.38	0.69	…
D	–0.16	0.39	–0.38	0.47	…
E	0.11	–0.01	–0.01	0.83	…
F	–0.54	0.37	0.49	0.77	…
1	0.19	0.16	–0.41	0.75	…
2	–0.10	0.17	0.03	0.96	…
3	–0.50	0.51	0.64	1.06	…

Table 2. Quantitative estimates for the symbol similarities

Symbol	1	2	3
A	0.4112	**0.9336**	0.6095
B	0.5235	0.4478	0.5770
C	**0.9638**	0.4399	0.1652
D	0.0460	0.5903	0.4516
E	0.4392	0.6639	0.3707
F	0.2212	0.6611	**0.9042**

Looking ahead, we note that such an invariant representation has been found. It can be expressed in terms of spectral analysis in the case of classical orthogonal expansion of functions as follows:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{N} a_n \cos nt + b_n \sin nt.$$

We emphasize the quantity $a_0/2$, which will be referred to as the level of function $f(t)$. This is a value that is independent of the phase shift and may be chosen as an invariant feature even at this stage. We obtain

$$\phi(t) = \sum_{n=1}^{N} a_n \cos nt + b_n \sin nt,$$

$$\psi(t) = \sum_{n=1}^{N} a'_n \cos nt + b'_n \sin nt.$$

Further, we assume that the exact relation

$$\phi(t) = \psi(t+\theta)$$

holds. Since

$$\psi(t+\theta) = \sum_{n=1}^{N} (a'_n \cos n(t+\theta) + b'_n \sin n(t+\theta)),$$

we obtain

$$a_n = a'_n \cos n\theta + b'_n \sin n\theta,$$

$$b_n = b'_n \cos n\theta - a'_n \sin n\theta.$$

Hence, we arrive at the relation

$$a_n^2 + b_n^2 = a_n'^2 + b_n'^2.$$

In other words, the set $\{(a_n, b_n)\}$ being considered as a matrix of vectors on the plane, the latter relation means that, regardless of possible phase shifts, the lengths of vectors $\{\overline{(a_n, b_n)}\}$ and $\{\overline{(a'_n, b'_n)}\}$ always coincide. Denote this quantity by

$$r_n = \sqrt{a_n^2 + b_n^2}.$$

It is clear that $\{r_n\}$ is a set of invariant features. Note however that it is insufficient to know values $\{r_n\}$ in order to reconstruct the initial function. For this purpose, we use the relation

$$a_n \cos nt + b_n \sin nt = r_n \cos(nt + \omega_n),$$

where

$$\omega_n = \arctan\frac{b_n}{a_n}.$$

As a result of phase shift, variable t being replaced with variable $t' = t + \theta$, all $\{\omega_n\}$ are also replaced by new values $\{\omega'_n\}$. From the last relation, we can see that

$$(\omega'_n - \omega_n) + n(t' - t) = 0.$$

This implies

$$\omega'_n = \omega_n - n\theta.$$

Continuing the analogy with vectors, we note that, in accordance with the expression obtained, the phase shift of any function does not change the length r_n of each vector $\{\overline{(a_n, b_n)}\}$ but changes its polar angle ω_n by a value, which is inverse and multiple of the phase change and proportional to the index of the vector. Representing vectors by diagrams, we see that, similarly to clock hands, they have different periods of rotation with respect to the phase variation: the speed of the first "hand" coincides with the speed of the phase variation, the rotation speed of the second hand is equal to the doubled speed of the first one, this of the third hand is equal to the tripled speed of the first one, etc. To verify the main conjecture that two arbitrary clock diagrams correspond to the same periodic function taken with different phases, it is sufficient to align their first hands: if the other hands also coincide, then the hypothesis is surely true; otherwise, it is false.[11] He have just obtained a formula for recalculating the new shifted values of $\{\omega'_n\}$.

To reduce the amount of calculations in practice, we may assume under some hypotheses that the first vectors on all clock-diagrams are directed in a certain way; for instance, the angle of the first hands is equal to zero

$$\omega_1 = 0,$$

of course if $r_1 \neq 0$. We will refer to such a process as a retrospect, and the value of θ used for this rotation will be called the retrospect depth (or simply depth). It is often useful to consider normalized representations of lengths $\{r'_n\}$

$$r'_n = \frac{r_n}{M},$$

where M is the magnitude

$$M^2 = \sum_{n=1}^{N} r_n^2.$$

The series of normalized retrospective coefficients $\{(r_n, \omega_n)\}$ will be referred to as the canonical form of the function.

It is clear that, knowing the level $a_0/2$, magnitude M, and depth θ of function $f(t)$, we can uniquely reconstruct it via its canonical form $\{(r_n, \omega_n)\}$ as follows:

$$f(t) = \frac{a_0}{2} + M\sum_{n=1}^{N} r_n \cos(n(t-\theta) + \omega_n).$$

By $T(f(t))$, we denote the operator associating with each function $f(t)$ its canonical form $\{(r_n, \omega_n)\}$. Then, we may assert that

$$T(f(t)) \equiv T(\alpha + \beta f(\{t+\gamma\}_T))$$

for any $\forall\alpha, \gamma \in \mathbb{R}, \beta \neq 0$.

This implies that function $f(t)$ may differ from its transform, the function $\alpha + \beta f(\{t + \gamma\}_T)$ only by three features, namely, the level, magnitude, and depth. As to the rest, these function have the identical analytical form, namely, the canonical form $\{(r_n, \omega_n)\}$. Therefore, we may consider that, under the aforementioned non-

[11] In the general case, if a function is equal to itself in period T/N, then $r_1 = r_2 = \ldots = r_{N-1} = 0$ and the Nth hand should be aligned.

strict assumptions, we have solved the problem of the start-point choice by proposing the method for constructing a representation invariant with respect to the phase shift. Below, we refer to this method as the method of clock-diagrams.

In the next section, we present an example of the use of the clock-diagram method in the problem of automatic visual orientation in the space.

SOLUTION OF A TYPICAL PROBLEM BY THE METHOD OF CLOCK-DIAGRAMS

In the section *Spectral Invariants of the Choice of the Start-Point. The Clock-Diagram Method*, we have demonstrated a method for recognition of periodic functions as functions of quite arbitrary nature. This method is very usefully applied to problems of contour recognition, where it is required to eliminate the uncertainty in the choice of the vectorization start-point. It is for solving this problem that this method was initially developed. Then, it was suggested testing the universality and efficiency of the developed apparatus in solution of another problem of pattern recognition, which is also connected with a problem of the type of the start-point choice, namely, the problem of automatic visual orientation in space with the help of the panoramic photography.

The statement of the problem is the following.

During the learning process, the recognition system is provided with a few photos made on a certain territory from different points. Each photo contains an image of a panoramic perspective (photo about 360°); moreover, the standpoint and orientation of shooting are determined for each shot.

During the recognition, the system should process a panoramic photo made by a mobile object from an unknown point with an unknown orientation of shooting and, possibly, with a high level of heterogeneous noise. The system must, in a short time, determine the location of the object as the most probable among all known locations and specify the orientation of shooting.

Since the topology of the image of the panoramic perspective is that of a cylinder, it is logical to consider values of colors along horizontal lines as periodic functions whose period is equal to the photo width. For simplicity of the recognition scheme being designed, we associate with each photo the periodic function whose values are arithmetic means of the brightness of all points of the image column with the same abscissa. This univariate periodic function will be referred to as the trace of the image. The further recognition process will be based on comparison of image traces.

As experimental data, we take a panoramic image made on a terminal of a seaport. We intend to learn the system to recognize pictures taken from different points. Moreover, we suppose that meteorological conditions may vary, and the photos may be significantly distorted by dust, glares, and other factors. All used photos are presented in Fig. 5. Under each photos, we present a strip of brightness that corresponds to values of the image trace. Applying operator $T(f(t))$ to the trace considered as a univariate periodic function $f(t)$, we obtain the canonical form presented as a clock-diagram for each photo.

Fig. 4. Tetouev's (Titúlany's) mark.

In the numerical experiment, the first three photos (A, B, and C) were entered into the system as learning objects, while the last three photos (1, 2, and 3) were entered as objects to be identified. On the basis of analysis of mutual correlations of the canonical forms, the system concluded correctly that the probability of the fact that photos (1, 3) were shot from a point located not far from the point corresponding to photo (A) is very high and the similar probability for the pair (2) and (C) is rather high.

Despite the high level of distortion of the images and displacement of the images with respect to one another, the system detected all real relations and made none erroneous conclusions, determined, with a high precision, the most probable point of shooting and the value of mutual disorientation in the photos. It used just the amount of computer time and memory that is necessary for generation of Table 3 composed of quantitative estimates for correlation of the first 20 coefficients of the canonical forms of the corresponding photos.

As a result of the numerical experiments, the high applicability of the derived estimates has been established to the solution of typical problems such as the automatic visual orientation in space of an object which can generate or scan around it a strip of brightness (illuminance) of the panorama.

Table 3. Quantitative estimates for the probabilities of the shooting points

Photo	1	2	3
A	**0.9987**	0.0392	**0.9998**
B	0.0612	0.2085	0.0589
C	0.2108	**0.8091**	0.1921

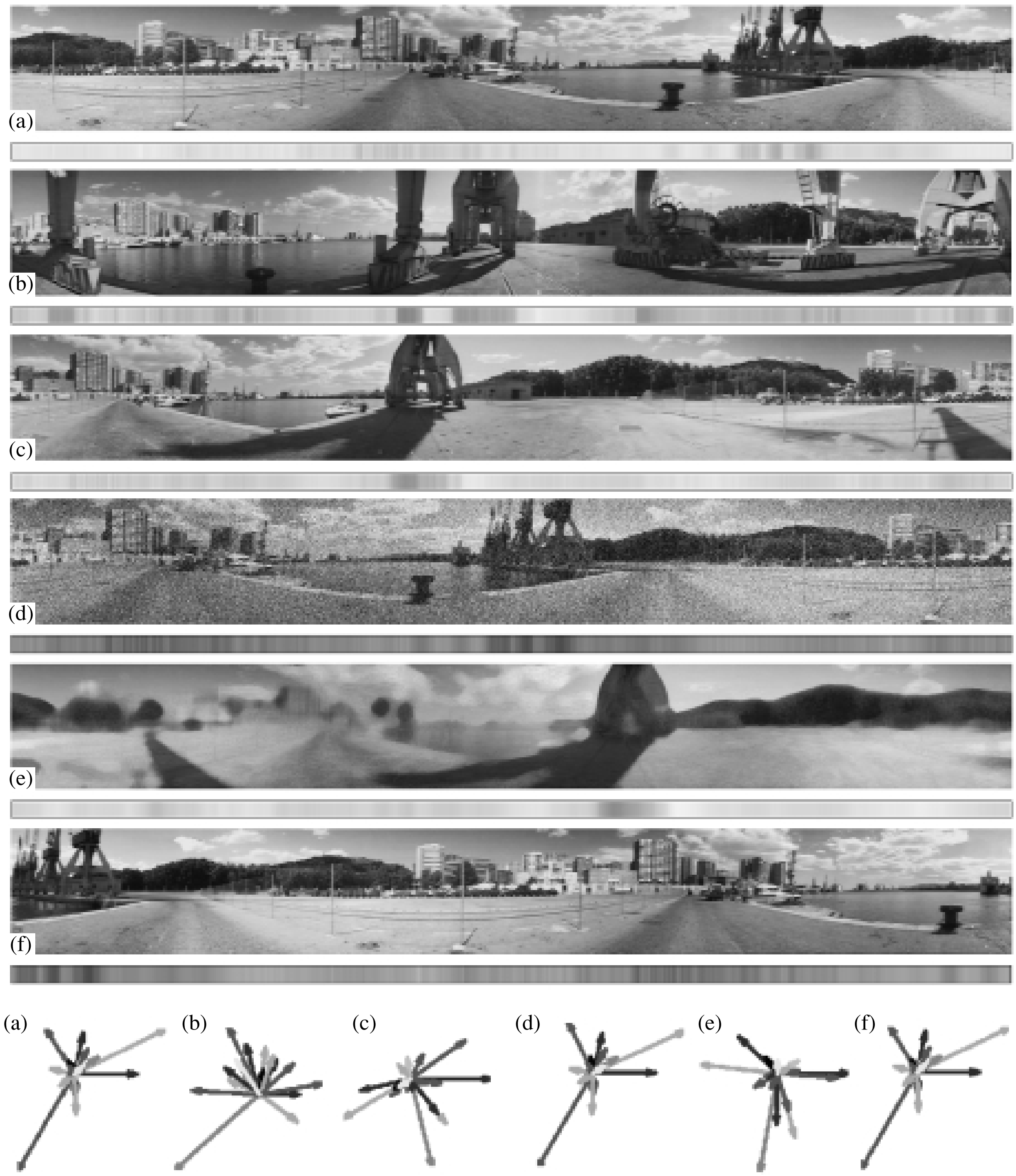

Fig. 5. Photos of the landscape panorama made from different points, as well as traces and clock-diagrams corresponding to these photos.

CONCLUSIONS

In the paper, we propose some methods for visual information processing, which are based on fundamental concepts of functional and statistical analysis. The adequacy of obtained numerical-analytical estimates demonstrates that there exists the possibility for solving even principally new problems (such as pattern recognition) by means of classical well-known concepts.

The rigorous mathematical concepts such as functional measure, approximation, and some other are, in fact, much less abstract than they were considered when investigations started. On contrary, licking all creation, the orthogonal expansions of functions detect within their theory quite adequate alternatives to rather complex concepts from the application domain. For instance, an "intelligent" machine provided with the "spectral" understanding of the geometrical shape cannot be confused by the phrase: "Masha's form is … how it can be explained, something average between Leili's and Natasha's forms." What will happen, and how these words will be understood by our "silicon intellectual"? Most probably, it will understand them literally and will directly average the corresponding values from the first and second series of coefficients. As a result, it will obtain the representation of a form (that is noticeable), which is really similar to both the previous forms in our human opinion.

Continuing these arguments, we would like to ask a series of questions. How can it be checked whether a given set of well-known forms is sufficient for description of an arbitrary person? How one can exclude forms useless for this description such as Masha "linearly dependent" of Leila and Natasha? To what degree such photo-robots are adequate? etc. To develop, for instance, artificial intelligence, it seems to be useful to construct in perspective an "orthogonal" theory of such nonstrict human concepts.

It is difficult to estimate the similarity of objects on the basis of only the similarity of their contours if the objects split into a group of contours. This forces us to create an algebra on the set of these groups in order to estimate adequately the relations and operations over them. For instance, retrieving a certain face in a file, we should pay attention to similarity of eye slits and, obviously, face profile rather than to coincidence of the form of lips which can be easily changed (face-painting, calogen, etc.). This results in necessity of creation of an evaluation function in the three-dimensional space ("lips," "eyes," "profile").

In conclusion, note that the proposed methods may be surely generalized to the multidimensional case and adapted to particular conditions of problems being solved. This is valid, for instance, if the clock-diagram method is used in the case of a bivariate function on a topology of a torus rather than that of a cylinder.

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